Drag force in a charged $\mathcal{N} = 4$ SYM plasma

Elena Cáceres

Facultad de Ciencias, Universidad de Colima,
Bernal Díaz del Castillo 340, Colima, México
E-mail: elenac@ucol.mx

Alberto Güijosa

Departamento de Física de Altas Energías,
Instituto de Ciencias Nucleares, Universidad Autónoma de México,
Apdo. Postal 70-543, D.F.04510, México
E-mail: alberto@nucleares.unam.mx

ABSTRACT: Following recent developments, we employ the AdS/CFT correspondence to determine the drag force exerted on an external quark that moves through an $\mathcal{N} = 4$ super-Yang-Mills plasma with a non-zero R-charge density (or, equivalently, a non-zero chemical potential). We find that the drag force is larger than in the case where the plasma is neutral, but the dependence on the charge is non-monotonic.

Contents

1. Introduction and summary

Central collisions of gold nuclei at RHIC are believed to produce the long-sought quark gluon plasma (QGP). RHIC experiments have found evidence of strong collective behavior, and the consensus is that we are dealing with a strongly-interacting liquid that can be modelled by hydrodynamics (see, e.g., [1] for a recent review). The hydrodynamic regime is completely characterized by transport coefficients (shear and bulk viscosity, etc.). RHIC data suggests that the QGP viscosity should be fairly small— the ratio of shear viscosity to entropy density has been estimated [2] to be $\eta/s \sim 0.1 - 0.2$, although uncertainties in this value are still quite large. Unfortunately, a calculation of the hydrodynamic coefficients from first principles, *i.e*, from the underlying microscopic theory, is currently out of our reach in the strong-coupling regime.

The gauge/gravity or AdS/CFT correspondence [3, 4] has been used to investigate observables in various interesting strongly-coupled gauge theories [5] where perturbation theory is not applicable. Policastro, Son and Starinets pioneered the study of hydrodynamic gauge theory properties using AdS/CFT [6, 7]. In [6], these authors computed the shear viscosity η of strongly-coupled $\mathcal{N}=4$ super-Yang-Mills (SYM) theory in $3+1$ dimensions and at finite temperature. They found that the ratio of shear viscosity to entropy density equals $1/4\pi$, a result that was later argued to be universal, in the sense that it applies to any gauge theory described by a supergravity dual, in the limit of large 't Hooft coupling [8].[1] These results raised the tantalizing possibility of using AdS/CFT to study the QGP. Transport coefficients of different thermal gauge theories have been calculated in [7]-[22]. Work attempting to narrow the gap between the gauge/gravity duality and RHIC may be found in [23, 24].

RHIC data also confirmed the existence of jet quenching in high-energy heavy ion collisions [25] and showed that the observed suppression of hadrons from fragmentation of hard partons is due to their interaction with the dense medium [26]. In a series of interesting and very recent papers, the AdS/CFT machinery has been brought to bear on the phenomenon of jet quenching and the associated parton energy loss. The authors

---

[1]The leading order correction in inverse powers of the 't Hooft coupling was determined in [9].

of [27] suggested that the jet-quenching parameter $\hat{q}$ used as a measure of energy loss in phenomenological models [28, 29] could be defined non-perturbatively in terms of a lightlike Wilson loop, and then employed the gauge/gravity duality to compute this loop in strongly-coupled $\mathcal{N} = 4$ SYM. The calculation was extended to the non-conformal case in [30]. The works [31, 32] studied the dynamics of moving strings in the AdS$_5$-Schwarzschild$\times$**S**^{5} background to determine the drag force that a hot neutral $\mathcal{N} = 4$ SYM plasma exerts on a moving quark. In [33], through an analysis of small string fluctuations, the authors determined the diffusion coefficient for the plasma, finding a result that is in agreement with [31].

In this note we extend the computation of [31, 32] to the near-horizon geometry associated with rotating near-extremal D3-branes, which allows us to examine how the drag force changes upon endowing the plasma with a non-zero charge (or, equivalently, chemical potential) under a U(1) subgroup of the SU(4) R-symmetry group. The fact that the QGP produced at RHIC is also found to carry a small but non-zero charge density (associated to baryon number) cannot but add interest to our calculation. But of course, since the string theory dual of QCD is not known, one cannot overemphasize that, for multiple reasons, comparison of AdS/CFT results with real data is, in general, a risky business (for discussion, see [31, 32]).

In section 2 we review the relevant properties of the rotating black three-brane geometry, and discuss the embedding of the Nambu-Goto string in this background. Section 3 contains the calculation of the drag force. The dependence of our general result (3.9) on the plasma temperature T and charge density J is shown in figure 1, obtained by solving the sixth-order equation (3.10) numerically. For small charge density, this equation can be solved perturbatively, and (3.13) gives the resulting drag force at next-to-leading order. Our main conclusion is that the force in the charged plasma case is *larger* than in the neutral case. Interestingly, we find the same result for the force in two cases where the external quark is coupled in different ways to the SYM scalar fields, despite the fact that in one ('polar') case the quark is neutral under the global U(1)$_R$, while in the other ('equatorial') case it is charged.

While the first version of this paper was in preparation an overlapping preprint [34] appeared. We will discuss its relation to our work at the end of section 3.

2. String in a rotating D3-brane background

The AdS/CFT correspondence [3, 4] equates the physics of an external quark in a finite-temperature *neutral* $\mathcal{N} = 4$ SYM plasma to that of a fundamental string that lives on the near-horizon geometry of a stack of static near-extremal D3-branes [35]. This connection was employed in [31, 32] to determine the drag force that the neutral plasma exerts on the quark. Our aim in this paper is to consider instead a plasma that is *charged* under the SU(4) R-symmetry, and so we must analyze a string that ploughs through a rotating D3-brane background. For simplicity we will restrict attention to the case where only one of the three SU(4) angular momenta is non-zero. The corresponding solution was obtained

in [36] (see also [37]);[2] we will follow here the conventions of [39]. In the near-horizon limit, the metric is

$$ds^2 = \frac{1}{\sqrt{H}}(-hdt^2 + d\vec{x}^2) + \sqrt{H}\left[\frac{dr^2}{\tilde{h}} - \frac{2lr_0^2}{R^2}\sin^2\theta dtd\phi \right. \tag{2.1}$$
$$\left. + r^2(\Delta d\theta^2 + \tilde{\Delta}\sin^2\theta d\phi^2 + \cos^2\theta d\Omega_3^2)\right],$$

where

$$H = \frac{R^4}{r^4\Delta},$$
$$\Delta = 1 + \frac{l^2\cos^2\theta}{r^2},$$
$$\tilde{\Delta} = 1 + \frac{l^2}{r^2},$$
$$h = 1 - \frac{r_0^4}{r^4\Delta},$$
$$\tilde{h} = \frac{1}{\Delta}\left(1 + \frac{l^2}{r^2} - \frac{r_0^4}{r^4}\right),$$
$$R^4 = 4\pi N g_s l_s^4 . \tag{2.2}$$

This geometry has an event horizon at the positive root of $\tilde{h}(r_H) = 0$,

$$r_H^2 = \frac{1}{2}\left(\sqrt{l^4 + 4r_0^4} - l^2\right), \tag{2.3}$$

and an associated Hawking temperature, angular momentum density, and angular velocity at the horizon

$$T = \frac{r_H}{2\pi R^2 r_0^2}\sqrt{l^4 + 4r_0^4}, \qquad J = \frac{lr_0^2 R^2}{64\pi^4 g_s^2 l_s^8}, \qquad \Omega = \frac{lr_H^2}{r_0^2 R^2}, \tag{2.4}$$

which translate respectively into the temperature, R-charge density and R-charge chemical potential in the gauge theory. For $J \neq 0$, there is an ergosphere between $r = r_H$ and the stationary limit surface $r = r_s(\theta)$ defined by $h(r_s, \theta) = 0$.

An external quark (a pointlike source in the fundamental representation of the SU(N) gauge group) that moves at constant velocity in the $x^1 \equiv x$ direction and carries a charge under the same global U(1)$_R$ $\subset$ SU(4)$_R$ as the plasma corresponds to a string whose embedding in the geometry (2.1) and in the static gauge $\sigma = r$, $\tau = t$ is of the general stationary form

$$X^1(r,t) = vt + \xi(r), \qquad \theta(r,t) = \nu t + \zeta(r), \qquad \phi(r,t) = \omega t + \chi(r), \tag{2.5}$$

with all other fields vanishing. As usual, the behavior at infinity describes the gauge theory in the extreme UV, where the pointlike quark is inserted, so the string must have a single

---

[2]The metric for the case with all three angular momenta turned on can be found in [38].

endpoint on the boundary. The corresponding boundary conditions[3]

$$X(r,t) \to X_\infty(t) \equiv vt, \quad \theta(r,t) \to \theta_\infty(t) \equiv \nu t + \zeta_\infty, \quad \phi(r,t) \to \phi_\infty(t) \equiv \omega t \quad \text{as} \quad r \to \infty,$$
(2.6)

specify both the trajectory of the quark (which determines its coupling $A_\mu(X_\infty(t))\partial_t X_\infty^\mu(t)$ to the gluonic fields,) and its global $U(1)_R$ charge (which controls its coupling $\Phi_i(X_\infty(t))\hat{n}_\infty^i(t)$ to the scalar fields, with $\hat{n}_\infty^i(t) \in \mathbf{R}^6$ the unit vector which points towards the north pole of the $\mathbf{S}^3 \subset \mathbf{S}^5$ lying at polar angle $\theta_\infty(t)$ and azimuthal angle $\phi_\infty(t)$ [41]). The tail of the string is associated instead with a flux tube that could be mapped out with the methods of [42, 43].[4] In the present case the tube which codifies a fixed gauge- and scalar-field pattern that follows the moving quark and becomes wider as one moves back along the $-x$ direction at a given time (or, equivalently, as time elapses at a fixed location in x) [32].

The equations of motion for the string are obtained from the Nambu-Goto Lagrangian

$$\mathcal{L} \equiv -\sqrt{-g} = -\sqrt{-\det(\partial_a X^\mu \partial_b X^\nu G_{\mu\nu})}\,,$$
(2.7)

where $G_{\mu\nu}$ ($\mu, \nu = 0, \ldots, 9$) is the spacetime metric (2.1) and g_{ab} the induced metric on the worldsheet. The stationary embedding (2.5) is such that $\partial_t(\partial\mathcal{L}/\partial\dot{X}^\mu) = 0 \,\forall\mu$, and as a result, the equations for the cyclic variables $X(r,t)$ and $\phi(r,t)$ amount to the statement that the conjugate spatial momentum densities

$$\pi_x \equiv \pi_x^r = \frac{\partial\mathcal{L}}{\partial X'}\,, \qquad \pi_\phi \equiv \pi_\phi^r = \frac{\partial\mathcal{L}}{\partial\phi'}$$
(2.8)

are constant. Our main goal is to determine the value of π_x, which controls the x-component of the force that each segment of the string exerts on its larger-r neighbor [31],

$$F_x = \frac{1}{2\pi\alpha'}\pi_x\,.$$
(2.9)

In the situation of interest the string trails behind its boundary endpoint (which is being pulled in the $+x$ direction by an external agent), so we should have $F_x < 0$ and therefore $\pi_x < 0$.

The equation of motion for $\theta(r,t)$, on the other hand, is rather complicated, due to fact that $\mathcal{L}$ has explicit θ-dependence. In particular, and in contrast with the non-rotating case, a constant-θ solution is possible for $l \neq 0$ only if $\theta_\infty = 0, \pi$ or $\theta_\infty = \pi/2$, just like in the $v = 0$ case studied previously in [40]. For simplicity, we will restrict attention to these cases, which describe a string that lies respectively perpendicular and parallel to the rotation plane of the black brane.

- In the 'polar' case $\theta_\infty = 0$ or π, the string points at constant angles $\theta(r,t) = 0, \pi$ and $\phi(r,t) = 0$, and reaches the horizon at the pole, where the ergosphere has vanishing

---

[3]Notice that an appropriate choice of origin allows us to set $\xi_\infty = 0 = \chi_\infty$ without loss of generality.

[4]After this work had appeared as a preprint on the arXiv, the relevant calculations were carried out both for the case of vanishing [44] and non-vanishing [45] R-charge density.

width ($\tilde{h} = h$, so the stationary limit surface coincides with the horizon). Starting from the definition of π_x in (2.8), it is then easy to infer that

$$X' = -\pi_x \sqrt{\frac{G_{rr}\left(-G_{tt}/G_{xx} - v^2\right)}{-G_{tt}(-G_{tt}G_{xx} - \pi_x^2)}} \, , \qquad (2.10)$$

where, in accord with the previous discussion, the overall sign has been chosen in such a way that $\pi_x < 0$ implies $X' > 0$, meaning that the string trails behind its boundary endpoint.

- In the 'equatorial' case $\theta_\infty = \pi/2$, the string is at constant polar angle $\theta(r,t) = \pi/2$ but can have a nontrivial azimuthal profile $\phi(r,t) = \omega t + \chi(r)$. One can again invert (2.8) to obtain X' and ϕ' in terms of $\pi_x, \pi_\phi, v, \omega$. To simplify the algebra, we will henceforth concentrate on the case $\omega = 0$, where the string does not rotate. This choice is clearly valid from the point of view of the gauge theory, where it amounts to specifying a time-independent coupling between the external quark and the scalar fields. On the other hand, it might seem counterintuitive from the gravity perspective, because we expect the string to penetrate beyond the ergosphere, where a point particle could not avoid rotating. Nevertheless, we will see in the next section that it is possible to find physical configurations where the string does precisely that. Setting $\omega = 0$, then, we find that the definitions (2.8) imply

$$X' = \sqrt{\frac{G_{rr}}{G_{xx}(G_{t\phi}^2 - G_{\phi\phi}G_{tt})D}} \left[\pi_\phi G_{t\phi}G_{xx}v + \pi_x \left\{-G_{t\phi}^2 + G_{\phi\phi}(G_{tt} + G_{xx}v^2)\right\}\right] \, ,$$

$$\phi' = \sqrt{\frac{G_{rr}G_{xx}}{(G_{t\phi}^2 - G_{\phi\phi}G_{tt})D}} \left[\pi_\phi G_{tt} + \pi_x G_{t\phi}v\right] \, , \qquad (2.11)$$

$$D \equiv 2\pi_\phi \pi_x G_{t\phi}G_{xx}v + \pi_x^2\{-G_{t\phi}^2 + G_{\phi\phi}(G_{tt} + G_{xx}v^2)\}$$
$$+ G_{xx}\{\pi_\phi^2 G_{tt} - (G_{t\phi}^2 - G_{\phi\phi}G_{tt})(G_{tt} + G_{xx}v^2)\} \, ,$$

where again the overall signs were chosen such that $\pi_x < 0$ for the desired trailing string.

3. Drag force in a charged plasma

We are now ready to compute the drag force exerted by the charged plasma on the quark, following [31, 32]. The first step is to demand that the solutions to (2.10) and (2.11) be well-defined over the entire region $r_H < r < \infty$. This condition is nontrivial because the solutions involve square roots of quantities that can become negative. As we will now learn, insisting that this does not happen will fix the values of π_x, π_ϕ. As in [31, 32], in the analysis to follow a special role will be played by the velocity-dependent radius $r_v(\theta)$ that is the largest root of the equation $h(r, \theta) - v^2 = 0$,

$$r_v(\theta)^2 = \frac{1}{2}\left(\sqrt{l^4 \cos^4 \theta + \frac{4r_0^4}{1 - v^2}} - l^2 \cos^2 \theta\right) \, . \qquad (3.1)$$

- In the polar case X' is given by equation (2.10). The numerator involves the factor $-G_{tt}/G_{xx} - v^2 = h - v^2$, which according to (3.1) changes sign at

$$r_v(0) = \sqrt{\frac{1}{2}\left(\sqrt{l^4 + \frac{4r_0^4}{1-v^2}} - l^2\right)} = r_v(\pi) . \tag{3.2}$$

The only way we can prevent X' from becoming imaginary for $r < r_v(0)$ is by choosing a value of π_x such that the denominator also changes sign at $r_v(0)$, i.e.

$$\pi_x = -\sqrt{\frac{h(r_v)}{H(r_v)}} = -\frac{r_0^2}{R^2}\frac{v}{\sqrt{1-v^2}} . \tag{3.3}$$

The string profile can then be determined by integrating the equation obtained by plugging (3.3) into (2.10),

$$X' = v\frac{r_0^2}{R^2}\frac{H}{h} = \frac{vr_0^2R^2}{r^4 + l^2r^2 - r_0^4} . \tag{3.4}$$

- In the equatorial case X' and ϕ' are given by (2.11). Using the explicit metric components (2.1), it is easy to convince oneself that, out of the various factors inside the square roots, only the function

$$D(r) = \left[r_0^4\pi_x^2 + \frac{r_0^8}{R^4}\right]r^{-2} + \left[\frac{r_0^4}{R^4}\pi_\phi^2 - 2vl\frac{r_0^2}{R^2}\pi_\phi\pi_x - (1-v^2)l^2\pi_x^2 - l^2\frac{r_0^4}{R^4}\right]r^0 \tag{3.5}$$

$$+ \left[-(1-v^2)\pi_x^2 - (2-v^2)\frac{r_0^4}{R^4}\right]r^2 + \left[(1-v^2)\frac{l^2}{R^4} - \frac{\pi_\phi^2}{R^4}\right]r^4 + \left[\frac{1-v^2}{R^4}\right]r^6$$

runs the risk of becoming negative. This function clearly approaches $+\infty$ both at $r \to 0$ and $r \to \infty$, so it must have at least one minimum at some intermediate point. Since $r^2D(r)$ is quartic in the variable r^2, it could in general have as many as four distinct roots at positive values of r. Numerical calculation shows that, for generic values of π_x, π_ϕ, the function $D(r)$ has exactly two positive roots, $r_1, r_2 > r_H$, and a single minimum in between. This implies that $D(r) < 0$ (and the solution (2.11) is ill-defined) in the intermediate range $r_1 < r < r_2$, an undesired feature that can only be avoided by choosing π_x, π_ϕ in such a way that the $D(r)$ curve has its minimum at $D(r_1 = r_2) = 0$. Analytically, this corresponds to demanding that the r^0 and r^4 terms in (3.5) vanish, implying that

$$\pi_\phi = l\sqrt{1-v^2} ,$$
$$\pi_x = -\frac{r_0^2}{R^2}\frac{v}{\sqrt{1-v^2}} , \tag{3.6}$$

after which D is found to take the form

$$D(r) = \frac{1-v^2}{R^4}\left[r^4 - \frac{r_0^4}{1-v^2}\right]^2 . \tag{3.7}$$

This function is manifestly non-negative, and evidently has a minimum and a double root at $r_0/(1-v^2)^{1/4}$. Notice that this is none other than the critical radius $r_v(\pi/2)$ defined in (3.1).

The string profile in this case follows from integration of the equations of motion (2.11), which simplify drastically after use of (3.6):

$$X' = \frac{vr_0^2 R^2}{r^4 + l^2 r^2 - r_0^4} \,, \tag{3.8}$$

$$\phi' = -\frac{lr^2}{r^4 + l^2 r^2 - r_0^4} \,.$$

These expressions are clearly well-defined over the entire range $r_H < r < \infty$. One might have also worried that the string worldsheet could become spacelike at some point inside the ergosphere, but in fact (3.8) leads to $\sqrt{-g} = \sqrt{1-v^2}$, which is manifestly real. So as promised, we have been able to find a physical configuration where the string penetrates into the ergosphere and yet does *not* rotate.[5] What has happened is that the string has managed to adopt an azimuthal profile $\phi(r)$ that leads to a precise balance between the string tensile force and the inertial drag due to the rotating geometry. Notice that, according to (3.6), this requires an external agent to pull on the boundary endpoint of the string, exerting the non-zero force π_ϕ. Notice also that the sign of ϕ' in (3.8) correctly reflects the fact that, as r increases, the string should wind in the direction opposite to the sense of rotation.

The second step is to use (3.3) or (3.6) in (2.9), to read off the force with which the string pulls back on its boundary endpoint, which by the AdS/CFT correspondence should be identified with the drag force that the charged plasma exerts on the moving quark. Surprisingly, the result is the same in the polar and equatorial cases,

$$F_x = \frac{dp_x}{dt} = -\frac{r_0^2/R^2}{2\pi l_s^2} \frac{v}{\sqrt{1-v^2}} \,. \tag{3.9}$$

We learn then that the value of the $\mathrm{U}(1)_R$ charge of the external quark affects the gluonic and scalar field distributions set up by the quark (which, as explained in section 2, are encoded in the shape of the string tail (3.4) or (3.8)), but does not alter the total drag force experienced by the quark. Notice that the end result (3.9) is identical to the one given in equation (12) of [32], except for the fact that r_H has been replaced here by r_0. The two radii indeed agree for the non-rotating case, but are related through (2.3) in the general case, which shows that the drag force *does* have an interesting dependence on the R-charge density of the plasma.

The third and final step is to rewrite (3.9) in terms of gauge theory parameters. Using (2.3) and (2.4) one finds that $\rho \equiv r_0^4/16\pi^4 R^8 T^4$ satisfies the sixth-order equation

$$16\rho^6 - \rho^5 - 4c^2\rho^4 + 8c^4\rho^3 - c^6\rho + c^8 = 0 \,, \tag{3.10}$$

---

[5]In the process, we have also learned that the solution ran the risk of being ill-defined not just inside the ergosphere, but in the region below the critical radius $r_v(\theta)$ defined in (3.1), which coincides with the stationary limit surface $r_s(\theta)$ only for $v = 0$.
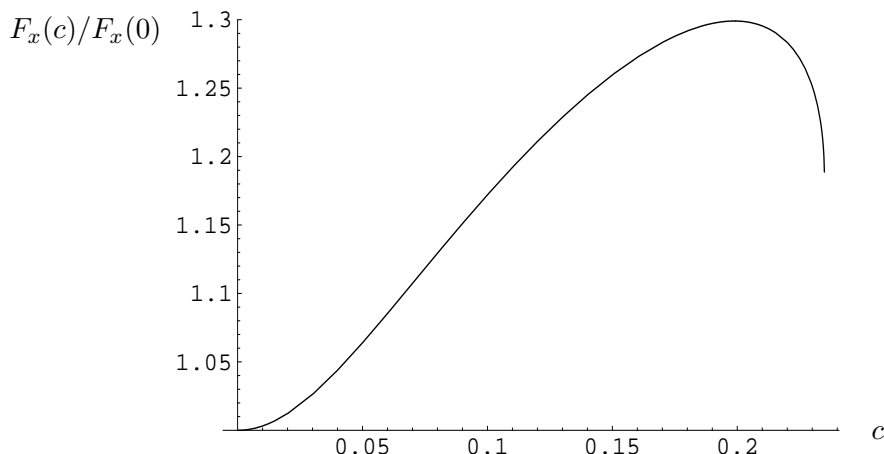
Figure 1: Drag force as a function of the dimensionless charge parameter $c = J/2\pi N^2 T^3$.

where we have defined a dimensionless charge parameter

$$c \equiv \frac{J}{2\pi N^2 T^3} \ . \tag{3.11}$$

In the neutral case $c = 0$, equation (3.10) implies that $\rho = 1/16$, from which one immediately recovers the well-known relation $r_0 = \pi R^2 T$. For small charge densities one can obtain r_0 by solving (3.10) in an expansion in powers of c. The result at next-to-leading order is

$$r_0 = \pi R^2 T \left[1 + \frac{4J^2}{\pi^2 N^4 T^6} + \mathcal{O}(J^4/N^8 T^{12}) \right] \ , \tag{3.12}$$

which together with (2.2), (3.9), $p_x = mv/\sqrt{1 - v^2}$ and $g_{\text{YM}}^2 \equiv 4\pi g_s$ leads to the final result

$$\frac{dp_x}{dt} = -\frac{\pi}{2} \sqrt{g_{\text{YM}}^2 N} T^2 \frac{p_x}{m} \left[1 + \frac{8J^2}{\pi^2 N^4 T^6} + \mathcal{O}(J^4/N^8 T^{12}) \right] \ . \tag{3.13}$$

From this we can read off the exponential relaxation time

$$\tau_0 = \frac{2}{\pi \sqrt{g_{\text{YM}}^2 N}} \frac{m}{T^2} \left[1 - \frac{8J^2}{\pi^2 N^4 T^6} + \mathcal{O}(J^4/N^8 T^{12}) \right] \ . \tag{3.14}$$

We learn here that turning on a nonzero R-charge density for the plasma *increases* the drag force exerted on the heavy quark (or equivalently, decreases its relaxation time). Notice that the J-dependent terms involve inverse powers of N, as has been found when computing other properties of the charged plasma (e.g., its entropy density [39]).

The behavior of the drag force for larger charge densities can be determined by solving (3.10) numerically. The result is shown in figure 1,[6] which displays the ratio between the force in the charged case and that in the neutral case. Real solutions exist only in the

---

[6]The original preprint that we posted to the arXiv gave the arbitrary-charge result only implicitly, through (3.10). We have added the plot to this later version in order to facilitate comparison with [34, 57].

finite range $0 \leq c \leq c_{\max} \simeq 0.2349$. The main feature is the maximum at $c_0 \simeq 0.199$, where the force ratio reaches a value $\simeq 1.299$, and beyond which it decreases to $\simeq 1.184$ at $c_{\max}$.

As explained in the Introduction, the authors of [27] advocated the use of a lightlike Wilson loop to define a parameter $\hat{q}$ meant to be an alternative characterization of energy loss in a thermal plasma. The works [58, 59, 57, 60] followed this procedure to determine $\hat{q}$ for the charged $\mathcal{N} = 4$ SYM plasma, obtaining results that are rather similar to the drag force computed above. The drag force curve given in figure 1 is in fact nearly identical to the analogous $\hat{q}$ curve depicted (as a function of $\xi \equiv 4\sqrt{2}c$) in figure 2 of [57]. (Particularly striking is the fact that the two ratios coincide at $\xi_{\max} = 4\sqrt{2}c_{\max}$.) This suggests that, at least in the current setting, both quantities indeed capture essentially the same physics.

It is also interesting to compare our results with those obtained in the closely related work [34], which appeared while the first version of this paper was in preparation. After carrying out a general drag force calculation applicable to any gauge theory whose dual gravity description involves an asymptotically AdS_{d+1} geometry, the author of [34] considers as a specific example the case of $d = 4$ R-charged $\mathcal{N} = 4$ SYM plasma, with all three of the independent charge parameters (corresponding to the independent rotations within $SU(4)_R$) turned on. In the present paper we have examined precisely this system, in the special subcase where two of these parameters are taken to vanish.

There is, however, an important difference in our calculations: whereas the string studied here moves in the ten-dimensional background (2.1), which is a solution of Type IIB supergravity describing the near-horizon geometry of a stack of rotating D3-branes, the string analyzed in [34] lives on the five-dimensional 'STU' background, which corresponds to a charged black hole solution of $\mathcal{N} = 8$ $D = 5$ gauged supergravity [61]. The latter theory is believed to be a consistent truncation of the Kaluza-Klein $\mathbf{S}^5$ reduction of Type IIB supergravity, and in [37] it was shown that the STU and spinning D3 backgrounds are indeed related in this manner: the background described by equations (5.1)-(5.5) of [34] is shown there to be a non-trivial truncation of our (2.1), with the identifications $\kappa_1 = (l/r_H)^2$, $\kappa_2 = \kappa_3 = 0$. The drag force deduced from the STU background in this case is given in equation (5.12) of [34], which is then to be compared with our spinning D3 result (3.9). The two forces are clearly *very* different: the result of [34] displays a complicated dependence on the velocity v and the charge parameter $\kappa = (l/r_H)^2$, which is completely unlike the simple dependence seen in (3.9).

This discrepancy seems to suggest that: i) the putative uplift from five to ten dimensions of the string configuration considered in [34] would have a complicated (but presumably still stationary, as in (2.5),) radial dependence for both the polar and the azimuthal angles, unlike the polar and equatorial profiles considered here; ii) string configurations more general than the ones considered here (e.g., a non-polar string dual to a quark whose coupling with the SYM scalar fields is time-independent) could potentially experience a drag force whose functional dependence on the relevant parameters is much more complicated than the one seen in (3.9). It would be interesting to explore this second point by searching for explicit configurations of this type in ten dimensions. The first point is obscured by the non-trivial nature of the truncation that connects the two backgrounds. In particular, given that the string of [34] does not couple to the five-dimensional gauge field

$A_t{}^\phi = -lr_0^2/R^2r^2\tilde{\Delta}$, which descends from the ten-dimensional metric component $G_t{}^\phi$ [37], it would seem like the angular dependence of its uplift should be special enough to somehow decouple the angular problem from the five-dimensional geometry, which is all that is captured by the Nambu-Goto action used in [34]. Our polar string seems to do precisely that, so it is not clear why this configuration appears not to be accessible from the five-dimensional perspective.

A second possibility is that the string of [34] is in some sense smeared over the $\mathbf{S}^5$. This would imply that its dual quark is qualitatively different from the one considered here, because its coupling to the SYM scalar fields would be somehow averaged over. A third possibility that should be kept in mind is that, even though the categorization of $\mathcal{N} = 8$ $D = 5$ supergravity as a consistent truncation of Type IIB *supergravity* would guarantee that any solution of the former can be uplifted to a solution of the latter, it could turn out not to be possible to capture the dynamics of a *string* that lives in the uplifted background in terms of a Nambu-Goto action that senses only the five-dimensional geometry.

The results obtained in [31, 32, 34] and the present paper refer to a colored object, the external quark. For comparison, it might be worth computing the drag force on a color-neutral object, such as a meson[7] [46, 41, 35] or a baryon [47 – 49]. We hope to return to this and related problems in future work.

Acknowledgments

References

[1] E.V. Shuryak, *What RHIC experiments and theory tell us about properties of quark-gluon plasma?*, *Nucl. Phys.* **A 750** (2005) 64 [`hep-ph/0405066`].

[2] D. Teaney, *Effect of shear viscosity on spectra, elliptic flow and Hanbury Brown-Twiss radii*, *Phys. Rev.* **D 68** (2003) 034913 [`nucl-th/0301099`].

[3] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 *Int. J. Theor. Phys.* **38** (1999) 1113 [`hep-th/9711200`].

---

[7]After this work had appeared as a preprint on the arXiv, the corresponding calculations were carried out in [50, 52, 51], and related work was reported in [53 – 56].

[4] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[5] C. Csáki, H. Ooguri, Y. Oz and J. Terning, *Glueball mass spectrum from supergravity*, *JHEP* **01** (1999) 017 [hep-th/9806021];
C. Csáki, Y. Oz, J. Russo and J. Terning, *Large-N QCD from rotating branes*, *Phys. Rev.* **D 59** (1999) 065012 [hep-th/9810186];
R.C. Brower, S.D. Mathur and C.-I. Tan, *Glueball spectrum for QCD from AdS supergravity duality*, *Nucl. Phys.* **B 587** (2000) 249 [hep-th/0003115];
E. Cáceres and R. Hernández, *Glueball masses for the deformed conifold theory*, *Phys. Lett.* **B 504** (2001) 64 [hep-th/0011204];
M. Krasnitz, *A two point function in a cascading N = 1 gauge theory from supergravity*, hep-th/0011179;
L.A. Pando Zayas, J. Sonnenschein and D. Vaman, *Regge trajectories revisited in the gauge/string correspondence*, *Nucl. Phys.* **B 682** (2004) 3 [hep-th/0311190];
X. Amador and E. Cáceres, *Spin two glueball mass and glueball Regge trajectory from supergravity*, *JHEP* **11** (2004) 022 [hep-th/0402061].

[6] G. Policastro, D.T. Son and A.O. Starinets, *The shear viscosity of strongly coupled N = 4 supersymmetric Yang-Mills plasma*, *Phys. Rev. Lett.* **87** (2001) 081601 [hep-th/0104066].

[7] G. Policastro, D.T. Son and A.O. Starinets, *From AdS/CFT correspondence to hydrodynamics*, *JHEP* **09** (2002) 043 [hep-th/0205052]; *From AdS/CFT correspondence to hydrodynamics. II: sound waves*, *JHEP* **12** (2002) 054 [hep-th/0210220].

[8] A. Buchel and J.T. Liu, *Universality of the shear viscosity in supergravity*, *Phys. Rev. Lett.* **93** (2004) 090602 [hep-th/0311175].

[9] A. Buchel, J.T. Liu and A.O. Starinets, *Coupling constant dependence of the shear viscosity in N = 4 supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 707** (2005) 56 [hep-th/0406264].

[10] P. Kovtun, D.T. Son and A.O. Starinets, *Viscosity in strongly interacting quantum field theories from black hole physics*, *Phys. Rev. Lett.* **94** (2005) 111601 [hep-th/0405231].

[11] A. Buchel, *N = 2* hydrodynamics*, *Nucl. Phys.* **B 708** (2005) 451 [hep-th/0406200].

[12] A. Parnachev and A. Starinets, *The silence of the little strings*, *JHEP* **10** (2005) 027 [hep-th/0506144].

[13] P. Benincasa, A. Buchel and A.O. Starinets, *Sound waves in strongly coupled non-conformal gauge theory plasma*, *Nucl. Phys.* **B 733** (2006) 160 [hep-th/0507026].

[14] A. Buchel, *Transport properties of cascading gauge theories*, *Phys. Rev.* **D 72** (2005) 106002 [hep-th/0509083].

[15] P. Benincasa and A. Buchel, *Transport properties of N = 4 supersymmetric Yang-Mills theory at finite coupling*, *JHEP* **01** (2006) 103 [hep-th/0510041].

[16] J. Mas, *Shear viscosity from R-charged AdS black holes*, *JHEP* **03** (2006) 016 [hep-th/0601144].

[17] D.T. Son and A.O. Starinets, *Hydrodynamics of R-charged black holes*, *JHEP* **03** (2006) 052 [hep-th/0601157].

[18] O. Saremi, *The viscosity bound conjecture and hydrodynamics of M2- brane theory at finite chemical potential, JHEP* **10** (2006) 083 [`hep-th/0601159`].

[19] K. Maeda, M. Natsuume and T. Okamura, *Viscosity of gauge theory plasma with a chemical potential from AdS/CFT, Phys. Rev.* **D 73** (2006) 066013 [`hep-th/0602010`].

[20] D. Teaney, *Finite temperature spectral densities of momentum and r- charge correlators in $N = 4$ Yang-Mills theory, Phys. Rev.* **D 74** (2006) 045025 [`hep-ph/0602044`].

[21] P. Kovtun and A. Starinets, *Thermal spectral functions of strongly coupled $N = 4$ supersymmetric Yang-Mills theory, Phys. Rev. Lett.* **96** (2006) 131601 [`hep-th/0602059`].

[22] P. Benincasa and A. Buchel, *Hydrodynamics of Sakai-Sugimoto model in the quenched approximation, Phys. Lett.* **B 640** (2006) 108 [`hep-th/0605076`].

[23] S.-J. Sin and I. Zahed, *Holography of radiation and jet quenching, Phys. Lett.* **B 608** (2005) 265 [`hep-th/0407215`].

[24] E. Shuryak, S.-J. Sin and I. Zahed, *A gravity dual of RHIC collisions*, `hep-th/0511199`.

[25] PHENIX collaboration, K. Adcox et al., *Suppression of hadrons with large transverse momentum in central Au + Au collisions at $\sqrt{S_{NN}} = 130\,Gev$, Phys. Rev. Lett.* **88** (2002) 022301 [`nucl-ex/0109003`].

[26] STAR collaboration, C. Adler et al., *Centrality dependence of high p_T hadron suppression in au + au collisions at $\sqrt{S_{NN}} = 130\,Gev$, Phys. Rev. Lett.* **89** (2002) 202301 [`nucl-ex/0206011`].

[27] H. Liu, K. Rajagopal and U.A. Wiedemann, *Calculating the jet quenching parameter from AdS/CFT*, `hep-ph/0605178`.

[28] R. Baier, D. Schiff and B.G. Zakharov, *Energy loss in perturbative QCD, Ann. Rev. Nucl. Part. Sci.* **50** (2000) 37–69 [`hep-ph/0002198`]. R. Baier, *Jet quenching, Nucl. Phys.* **A 715** (2003) 209 [`hep-ph/0209038`].

[29] A. Kovner and U.A. Wiedemann, *Gluon radiation and parton energy loss*, `hep-ph/0304151`.

[30] A. Buchel, *On jet quenching parameters in strongly coupled non- conformal gauge theories, Phys. Rev.* **D 74** (2006) 046006 [`hep-th/0605178`].

[31] C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz and L.G. Yaffe, *Energy loss of a heavy quark moving through $N = 4$ supersymmetric Yang-Mills plasma, JHEP* **07** (2006) 013 [`hep-th/0605158`].

[32] S.S. Gubser, *Drag force in AdS/CFT*, `hep-th/0605182`.

[33] J. Casalderrey-Solana and D. Teaney, *Heavy quark diffusion in strongly coupled $N = 4$ Yang-Mills, Phys. Rev.* **D 74** (2006) 085012 [`hep-ph/0605199`].

[34] C.P. Herzog, *Energy loss of heavy quarks from asymptotically AdS geometries, JHEP* **09** (2006) 032 [`hep-th/0605191`].

[35] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories, Adv. Theor. Math. Phys.* **2** (1998) 505 [`hep-th/9803131`].

[36] J.G. Russo, *New compactifications of supergravities and large-N QCD, Nucl. Phys.* **B 543** (1999) 183 [`hep-th/9808117`].

[37] M. Cvetič and S.S. Gubser, *Phases of r-charged black holes, spinning branes and strongly coupled gauge theories*, *JHEP* **04** (1999) 024 [hep-th/9902195].

[38] P. Kraus, F. Larsen and S.P. Trivedi, *The coulomb branch of gauge theory from rotating branes*, *JHEP* **03** (1999) 003 [hep-th/9811120].

[39] S.S. Gubser, *Thermodynamics of spinning D3-branes*, *Nucl. Phys.* **B 551** (1999) 667 [hep-th/9810225].

[40] A. Brandhuber and K. Sfetsos, *Wilson loops from multicentre and rotating branes, mass gaps and phase structure in gauge theories*, *Adv. Theor. Math. Phys.* **3** (1999) 851 [hep-th/9906201].

[41] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002].

[42] U.H. Danielsson, E. Keski-Vakkuri and M. Kruczenski, *Vacua, propagators and holographic probes in AdS/CFT*, *JHEP* **01** (1999) 002 [hep-th/9812007].

[43] C.G. Callan Jr. and A. Güijosa, *Undulating strings and gauge theory waves*, *Nucl. Phys.* **B 565** (2000) 157 [hep-th/9906153].

[44] J.J. Friess, S.S. Gubser and G. Michalogiorgakis, *Dissipation from a heavy quark moving through N = 4 super- Yang-Mills plasma*, *JHEP* **09** (2006) 072 [hep-th/0605292];
J.J. Friess, S.S. Gubser, G. Michalogiorgakis and S.S. Pufu, *The stress tensor of a quark moving through N = 4 thermal plasma*, hep-th/0607022.

[45] Y.-h. Gao, W.-s. Xu and D.-f. Zeng, *Wake of color fileds in charged N = 4 SYM plasmas*, hep-th/0606266.

[46] S.-J. Rey and J.-T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [hep-th/9803001].

[47] Y. Imamura, *Supersymmetries and BPS configurations on anti-de Sitter space*, *Nucl. Phys.* **B 537** (1999) 184 [hep-th/9807179].

[48] C.G. Callan Jr., A. Güijosa and K.G. Savvidy, *Baryons and string creation from the fivebrane worldvolume action*, *Nucl. Phys.* **B 547** (1999) 127 [hep-th/9810092].

[49] C.G. Callan Jr., A. Güijosa, K.G. Savvidy and O. Tafjord, *Baryons and flux tubes in confining gauge theories from brane actions*, *Nucl. Phys.* **B 555** (1999) 183 [hep-th/9902197].

[50] K. Peeters, J. Sonnenschein and M. Zamaklar, *Holographic melting and related properties of mesons in a quark gluon plasma*, *Phys. Rev.* **D 74** (2006) 106008 [hep-th/0606195].

[51] M. Chernicoff, J.A. García and A. Güijosa, *The energy of a moving quark-antiquark pair in an N = 4 sym plasma*, *JHEP* **09** (2006) 068 [hep-th/0607089].

[52] H. Liu, K. Rajagopal and U.A. Wiedemann, *An AdS/CFT calculation of screening in a hot wind*, hep-ph/0607062.

[53] E. Cáceres, M. Natsuume and T. Okamura, *Screening length in plasma winds*, *JHEP* **10** (2006) 011 [hep-th/0607233].

[54] P.C. Argyres, M. Edalati and J.F. Vázquez-Poritz, *No-drag string configurations for steadily moving quark- antiquark pairs in a thermal bath*, hep-th/0608118.

[55] S.D. Avramis, K. Sfetsos and D. Zoakos, *On the velocity and chemical-potential dependence of the heavy-quark interaction in N = 4 SYM plasmas*, `hep-th/0609079`.

[56] J.J. Friess, S.S. Gubser, G. Michalogiorgakis and S.S. Pufu, *Stability of strings binding heavy-quark mesons*, `hep-th/0609137`.

[57] S.D. Avramis and K. Sfetsos, *Supergravity and the jet quenching parameter in the presence of R-charge densities*, `hep-th/0606190`.

[58] E. Cáceres and A. Güijosa, *On drag forces and jet quenching in strongly coupled plasmas*, `hep-th/0606134`.

[59] F.-L. Lin and T. Matsuo, *Jet quenching parameter in medium with chemical potential from AdS/CFT*, *Phys. Lett.* **B 641** (2006) 45 [`hep-th/0606136`].

[60] N. Armesto, J.D. Edelstein and J. Mas, *Jet quenching at finite 't Hooft coupling and chemical potential from AdS/CFT*, *JHEP* **09** (2006) 039 [`hep-ph/0606245`].

[61] K. Behrndt, M. Cvetič and W.A. Sabra, *Non-extreme black holes of five dimensional N = 2 AdS supergravity*, *Nucl. Phys.* **B 553** (1999) 317 [`hep-th/9810227`].